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Soft Symmetric Difference Complement-Lambda Product of Groups

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Abstract

The present study introduces the soft symmetric difference complement–lambda product of soft sets whose parameter sets are group. The key algebraic properties are investigted, its algebraic characteristics is analyzed in relation to identity, absorbing elements, null and absolute soft sets. Its broad usefulness in abstract algebraic modeling are further demonstrated by its smooth integration into soft inclusion hierarchies with generalized soft equalities.

Keywords: Soft sets, Soft subsets, Soft equalities, Soft symmetric difference complement-lambda.

1 | Introduction

Mathematical models have been developed to capture uncertainty; however traditional approaches like fuzzy sets [1] face limitations due to subjective assumptions. Soft set theory, introduced by Molodtsov [2] overcomes these constraints, and it has been further extended by Maji et al. [3] and Pei and Miao [4]. Subsequent extensions by Ali et al. [5] introduced restricted and extended operations. Numerous studies [6–19] have since advanced the field by proposing new binary operations.

Recently, soft set theory's algebraic structure has grown substantially through the binary operations [20-33] and the notions of subsethood and equality. Foundational work by Pei and Miao [4], Feng et al. [34], and Qin and Hong [35] further elaborated by Jun and Yang [36] and Liu et al. [37]. Feng and Li [38] demonstrated that certain quotient structures are semigroups. Generalizations such as g-soft, gf-soft, and T-soft equalities, introduced by Abbas et al. [39], [40], Al-shami [41], and Alshami and El-Shafei [42], incorporated congruence

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and lattice-theoretic theories, alongside dual soft union–intersection products within analogous structures [43–46].

The current study proposes the soft symmetric difference complement perspectives. Çağman and Enginoğlu [47] rectified foundational inconsistencies, establishing a logically consistent. Building on this foundation, soft intersection–union products were applied to ring [46], semigroup [48], and group–lambda product defined on soft sets with parameters of groups. Developed axiomatically, the operation undergoes thorough algebraic scrutiny, its identity elements, null and absolute soft sets, and absorbing elements are rigorously examined. This establishes a basis for generalized soft group theory with potential applications in abstract algebra and decision-making. Section 2 introduces formal definitions essential for theoretical development. Section 3 defines the soft symmetric difference complement–lambda product and its comprehensive algebraic theory is conducted in detail. Section 4 presents key conclusions and outlines prospective extensions for soft algebra.

2 | Preliminaries

The present work is fully situated within the axiomatic reformulation by Çağman and Enginoğlu [47], which serves as the foundational basis for all subsequent definitions, operations, and algebraic developments presented herein.

Definition 1 ([47]). Let E be a parameter set, U be a universal set, $\mathrm{P(U)}$ be the power set of U, and $\mathcal{H} \subseteq \mathrm{E}$. Then, the soft set $\mathfrak{I}_{\mathcal{H}}$ over U is a function such that $\mathfrak{I}_{\mathcal{H}}\colon \mathrm{E} \to \mathrm{P(U)}$, where for all $\mathrm{w} \notin \mathcal{H}$, $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \emptyset$. That is,

$$\mathfrak{I}_{\mathcal{H}} = \{(\mathrm{w}.\mathfrak{I}_{\mathcal{H}}(\mathrm{w}))\colon \mathrm{w} \in \mathrm{E}\}.$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2 ([47]). Let $\mathfrak{I}_{\mathcal{H}}$ be an $\mathcal{SS}$. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \emptyset$ for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_{\mathrm{E}}$, and if $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \mathrm{U}$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called an absolute SS and indicated by U_{E}.

Definition 3 ([47]). Let $\mathfrak{I}_{\mathcal{H}}$ and $\mathfrak{q}_{\aleph}$ be two $\mathcal{SS}$s. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) \subseteq \mathfrak{q}_{\aleph}(\mathrm{w})$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is a soft subset of $\mathfrak{q}_{\aleph}$ and indicated by $\mathfrak{I}_{\mathcal{H}} \tilde{\subseteq} \mathfrak{q}_{\aleph}$. If $\mathfrak{I}_{\mathcal{H}}(\mathrm{w}) = \mathfrak{q}_{\aleph}(\mathrm{w})$, for all $\mathrm{w} \in \mathrm{E}$, then $\mathfrak{I}_{\mathcal{H}}$ is called soft equal to $\mathfrak{q}_{\aleph}$, and denoted by $\mathfrak{I}_{\mathcal{H}} = \mathfrak{q}_{\aleph}$.

Definition 4 ([47]). Let $\mathrm{f}_{\mathcal{H}}$ be an $\mathcal{SS}$, Then, the complement of $\mathrm{f}_{\mathcal{H}}$ denoted by ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}$, is defined by the soft set ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}\colon \mathrm{E} \to \mathrm{P(U)}$ such that ${\mathrm{f}_{\mathcal{H}}}^{\mathrm{c}}(\mathrm{e}) = \mathrm{U}\backslash\mathrm{f}_{\mathcal{H}}(\mathrm{e}) = (\mathrm{f}_{\mathcal{H}}(\mathrm{e}))'$, for all $\mathrm{e} \in \mathrm{E}$.

Definition 5 ([47]). Let $\mathfrak{I}_{\mathcal{H}}$ and $\mathfrak{q}_{\aleph}$ be two $\mathcal{SS}$s. Then, the symmetric difference of $\mathfrak{I}_{\mathcal{H}}$ and $\mathfrak{q}_{\aleph}$ is the $\mathcal{SS}$ $\mathfrak{I}_{\mathcal{H}}\tilde{\Delta}\mathfrak{q}_{\aleph}$, where $(\mathfrak{I}_{\mathcal{H}}\tilde{\Delta}\mathfrak{q}_{\aleph})(\mathrm{w}) = \mathfrak{I}_{\mathcal{H}}(\mathrm{w})\Delta\mathfrak{q}_{\aleph}(\mathrm{w})$, for all $\mathrm{w} \in \mathrm{E}$.

Definition 6 ([49]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\mathfrak{q}_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft S-subset of $\mathfrak{q}_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{S}} \mathfrak{q}_{\aleph}$ if for all $\mathrm{w} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(\mathrm{w}) = \mathcal{M}$ and $\mathfrak{q}_{\aleph}(\mathrm{w}) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs $\mathfrak{I}_{\mathrm{K}}$ and $\mathfrak{q}_{\aleph}$ are said to be soft S-equal, denoted by $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} \mathfrak{q}_{\aleph}$, if $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{S}} \mathfrak{q}_{\aleph}$ and $\mathfrak{q}_{\aleph} \tilde{\subseteq}_{\mathrm{S}} \mathfrak{I}_{\mathrm{K}}$.

It is obvious that if $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} \mathfrak{q}_{\aleph}$, then $\mathfrak{I}_{\mathrm{K}}$ and $\mathfrak{q}_{\aleph}$ are the same constant functions, that is, for all $\mathrm{w} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(\mathrm{w})= \mathfrak{q}_{\aleph}(\mathrm{w}) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set .

Definition 7 ([49]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\mathfrak{q}_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft A-subset of $\mathfrak{q}_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} \tilde{\subseteq}_{\mathrm{A}} \mathfrak{q}_{\aleph}$, if, for each $a.\mathscr{b} \in \mathrm{E}$, $\mathfrak{I}_{\mathrm{K}}(a) \subseteq \mathfrak{q}_{\aleph}(\mathscr{b})$.

Definition 8 ([49]). Let $\mathfrak{I}_{\mathrm{K}}$ and $\mathfrak{q}_{\aleph}$ be two $\mathcal{SS}$s. Then, $\mathfrak{I}_{\mathrm{K}}$ is called a soft S-complement of $\mathfrak{q}_{\aleph}$, denoted by $\mathfrak{I}_{\mathrm{K}} =_{\mathrm{S}} (\mathfrak{q}_{\aleph})^{\mathrm{c}}$, if, for all $\mathrm{w} \in \mathrm{E}$. $\mathfrak{I}_{\mathrm{K}}(\mathrm{w}) = \mathcal{M}$ and $\mathfrak{q}_{\aleph}(\mathrm{w}) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = \mathrm{U}\backslash\mathcal{D}$.

For additional information on SSs, we refer to [50-73].

From now on, let G be a group, and $\mathrm{S_G(U)}$ denotes the collection of all $\mathcal{SS}$s over U, whose parameter sets are G; that is, each element of $\mathrm{S_G(U)}$ is an $\mathcal{SS}$ parameterized by G. Moreover, let Δ represent the classical symmetric difference operation. Then, the symmetric difference of the family $\mathfrak{B} = \{C_i: i \in I\}$ such that I is an index set, is denoted by

$$\triangle \mathfrak{B} = \underset{i\in I}{\triangle} C_i = C_1 \Delta C_2 \Delta \dots \Delta C_n.$$

Definition 9 ([74]). Let f_G and g_G be two SSs. Then, the soft symmetric difference-gamma product $f_G \otimes_{s/g} g_G$ is defined by

$$(f_G \otimes_{s/g} g_G)(x) = \underset{x=yz}{\triangle}(f_G(y) \gamma g_G(z)) = \underset{x=yz}{\triangle}((f_G(y)') \cap g_G(z)), \quad y, z \in G.$$

3 | Soft Symmetric Difference Complement-Lambda Product of Groups

This section presents a comprehensive algebraic analysis of the soft symmetric difference complement–lambda product, a novel binary operation defined on soft sets whose parameter domains possess inherent group-theoretic structure. From now on, the symmetric difference complement of the family $\mathfrak{B} = \{C_i: i \in I\}$ such that I is an index set, is denoted by

$$\coprod \mathfrak{B} = \coprod_{i\in I} C_i = (C_1 \Delta C_2 \Delta \dots \Delta C_n)'.$$

Definition 10. Let $\mathfrak{I}_G$ and q_G be two SSs. Then, the soft symmetric difference complement-lambda product $\mathfrak{I}_G \otimes_{s'/l} q_G$ is defined by

$$(\mathfrak{I}_G \otimes_{s'/l} q_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \lambda q_G(\eta)) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup q_G{}^c(\eta)), \varrho, \eta \in G,$$

for all $\varphi \in G$. For more lambda (λ) operation of sets, we refer to [26].

Note: the soft symmetric difference complement-lambda product is well-defined in $S_G(U)$. In fact, let $\mathfrak{I}_G$, q_G, m_G, $k_G \in S_G(U)$ such that $(\mathfrak{I}_G, q_G) = (m_G, k_G)$. Then, $\mathfrak{I}_G = m_G$ and $q_G = k_G$, implying that $\mathfrak{I}_G(\varphi) = m_G(\varphi)$ and $q_G(\varphi) = k_G(\varphi)$, for all $\varphi \in G$. Thereby, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} q_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup q_G{}^c(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (m_G(\varrho) \cup k_G{}^c(\eta))$$

$$= (m_G \otimes_{s'/l} k_G)(\varphi).$$

Hence, $\mathfrak{I}_G \otimes_{s'/l} q_G = m_G \otimes_{s'/l} k_G$.

Example 1. Consider the group $G = \{\sigma. \rho\}$ with the following operation:

Table 1. Definition of group $G = \{o, \rho\}$ with a specified operation.

0	σ	ρ
σ	σ	ρ
ρ	ρ	σ

Let $\mathfrak{I}_G$ and q_G be two SSs over $U = D_2 = \{< x. y >: x^2 = y^2 = e. xy = yx\} = \{e. x. y. yx\}$ as follows:

$\mathfrak{I}_G = \{(\sigma. \{x. yx\}). (\rho. \{e. x. y\})\}$ and $q_G = \{(\sigma. \{e. y. yx\}). (\rho. \{e. x\})\}$.

Since $\sigma = \sigma\sigma = \rho\rho$, $(\Im_G \otimes_{s'/l} q_G)(\sigma) = \left(\left(\Im_G(\sigma) \cup q_G{}^c(\sigma)\right)\Delta\left(\Im_G(\rho) \cup q_G{}^c(\rho)\right)\right)' = \{x, yx\}$, and since $\rho = \sigma\rho = \rho\sigma$, $(\Im_G \otimes_{s'/l} q_G)(\rho) = \left(\left(\Im_G(\sigma) \cup q_G{}^c(\rho)\right)\Delta\left(\Im_G(\rho) \cup q_G{}^c(\sigma)\right)\right)' = \{x, y\}$ is obtained. Hence,

$$\Im_G \otimes_{s'/l} q_G = \{(\sigma, \{x, yx\}), (\rho, \{x, y\})\}.$$

Proposition 1. The set $S_G(U)$ is closed under the soft symmetric difference complement-lambda product. That is, if $\Im_G$ and q_G are two $\mathcal{SS}$s, then so is $\Im_G \otimes_{s'/l} q_G$.

Proof: it is obvious that the soft symmetric difference complement-lambda product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft symmetric difference complement-lambda product.

Proposition 2. The soft symmetric difference complement-lambda product is not associative in $S_G(U)$.

Proof: consider the group G and the $\mathcal{SS}$s $\Im_G$ and q_G in *Example 1*. Let λ_G be an $\mathcal{SS}$ over $U = \{e. x. y. yx\}$ such that $\lambda_G = \{(\sigma, \{x, yx\}), (\rho, \{y\})\}$. Since $\Im_G \otimes_{s'/l} q_G = \{(\sigma, \{x, yx\}), (\rho, \{x, y\})\}$, then,

$$(\Im_G \otimes_{s'/l} q_G) \otimes_{s'/l} \lambda_G = \{(\sigma, U), (\rho, \{e, x\})\}.$$

Moreover, since $q_G \otimes_{s'/l} \lambda_G = \{(\sigma, \{e, yx\}), (\rho, \{e, x, y\})\}$, then

$$\Im_G \otimes_{s'/l} (q_G \otimes_{s'/l} \lambda_G) = \{(\sigma, \{x, y, yx\}), (\rho, \{x\})\}.$$

Thereby, $(\Im_G \otimes_{s'/l} q_G) \otimes_{s'/l} \lambda_G \neq \Im_G \otimes_{s'/l} (q_G \otimes_{s'/l} \lambda_G)$.

Proposition 3. The soft symmetric difference complement-lambda product is not commutative in $S_G(U)$.

Proof: consider the $\mathcal{SS}$s $\Im_G$ and q_G over $U = \{e, x, y, yx\}$ in *Example 1*. Then,

$$\Im_G \otimes_{s'/l} q_G = \{(\sigma. \{x. yx\}) . (\rho. \{x. y\})\},$$

and

$$q_G \otimes_{s'/l} \Im_G = \{(\sigma. \{e. yx\}) . (\rho. \{e. y\})\},$$

İmplying that $\Im_G \otimes_{s'/l} q_G \neq q_G \otimes_{s'/l} \Im_G$.

Proposition 4. The soft symmetric difference complement-lambda product is not idempotent in $S_G(U)$.

Proof: consider the $\mathcal{SS}$ $\Im_G$ in *Example 1*. Then,

$$\Im_G \otimes_{s'/l} \Im_G = \{(\sigma, U), (\rho, \{x\})\}$$

Implying that $\Im_G \otimes_{s'/l} \Im_G \neq \Im_G$.

Proposition 5. Let $\Im_G$ be a constant $\mathcal{SS}$. Then,

I. $\Im_G \otimes_{s'/l} \Im_G = \emptyset_G$, where $|G| = \gamma$ and γ is a positive odd integer.

II. $\Im_G \otimes_{s'/l} \Im_G = U_G$, where $|G| = \gamma$ and γ is a positive even integer.

Proof: let $\Im_G$ be a constant $\mathcal{SS}$ such that, for all $\varphi \in G$, $\Im_G(\varphi) = N$, where N is a fixed set.

I. Let $|G| = \gamma$, where γ is a positive odd integer. Then, for all $\varphi \in G$,

$$(\Im_G \otimes_{s'/l} \Im_G)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\Im_G(\varrho) \cup \Im_G{}^c(\eta)\right) = \underbrace{(U\Delta U\Delta \ldots \Delta U)'}_{\gamma \text{ times } U, \text{ where } \gamma \text{ is odd}} = \emptyset_G(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} \Im_G = \emptyset_G$.

II. Let $|G| = \gamma$, where γ is a positive even integer. Then, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{I}_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \mathfrak{I}_G{}^c(\eta)) = \underbrace{(U\Delta U\Delta \ldots \Delta U)'}_{ꝗ \text{ times } U, \text{ where } ꝗ \text{ is even}} = U_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{I}_G = U_G$.

Remark 1. Let $S_G{}^*(U)$ be the collection of all constant SS. Then, the soft symmetric difference complement-lambda product is not idempotent in $S_G{}^*(U)$ either.

Proposition 6. Let $\mathfrak{I}_G$ be an SS. Then,

I. $U_G \otimes_{s'/l} \mathfrak{I}_G = \emptyset_G$, where $|G| = ꝗ$ and $ꝗ$ is a positive odd integer.

II. $U_G \otimes_{s'/l} \mathfrak{I}_G = U_G$, where $|G| = ꝗ$ and $ꝗ$ is a positive even integer.

Proof: let $\mathfrak{I}_G$ be an SS.

I. Let $\varphi \in G$ and $|G| = ꝗ$, where $ꝗ$ is a positive odd integer. Then, for all $\varphi \in G$,

$$(U_G \otimes_{s'/l} \mathfrak{I}_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (U_G(\varrho) \cup \mathfrak{I}_G{}^c(\eta)) = \coprod_{\varphi=\varrho\eta} (U \cup \mathfrak{I}_G{}^c(\eta)) = \emptyset_G(\varphi).$$

Thus, $U_G \otimes_{s'/l} \mathfrak{I}_G = \emptyset_G$.

II. Let $\varphi \in G$ and $|G| = ꝗ$, where $ꝗ$ is a positive even integer. Then, for all $\varphi \in G$,

$$(U_G \otimes_{s'/l} \mathfrak{I}_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (U_G(\varrho) \cup \mathfrak{I}_G{}^c(\eta)) = \coprod_{\varphi=\varrho\eta} (U \cup \mathfrak{I}_G{}^c(\eta)) = U_G(\varphi).$$

Thus, $U_G \otimes_{s'/l} \mathfrak{I}_G = U_G$.

Remark 2. U_G is the left absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ꝗ$ and $ꝗ$ is a positive even integer by *Proposition 6* (II).

Note: U_G is not the right absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ꝗ$ and $ꝗ$ is a positive even integer. In fact, consider the SS $\mathfrak{I}_G$ in *Example 1*. Then,

$$\mathfrak{I}_G \otimes_{s'/l} U_G = \{(\sigma, \{x\}), (\rho, \{x\})\} \neq U_G.$$

Remark 3. U_G is not the absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ꝗ$ and $ꝗ$ is a positive even integer.

Proposition 7. Let $\mathfrak{I}_G$ be a constant SS. Then,

I. $\mathfrak{I}_G \otimes_{s'/l} U_G = \mathfrak{I}_G{}^c$, where $|G| = ꝗ$ and $ꝗ$ is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/l} U_G = U_G$, where $|G| = ꝗ$ and $ꝗ$ is a positive even integer.

Proof: let $\mathfrak{I}_G$ be a constant SS such that, for all $\varphi \in G$, $\mathfrak{I}_G(\varphi) = \mathrm{N}$.

I. Let $|G| = ꝗ$, where $ꝗ$ is a positive odd integer. Then, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} U_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U_G{}^c(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset_G(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset)$$

$= \mathfrak{I}_G{}^c(\varphi),$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} U_G = \mathfrak{I}_G{}^c$.

II. Let $|G| = \text{ɏ}$, where ɏ is a positive even integer. Then, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} U_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U_G{}^c(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset_G(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset)$$

$$= U_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} U_G = U_G$.

Remark 4. U_G is the absorbing element of the soft symmetric difference complement-lambda product in $S_G{}^*(U)$, where $|G| = \text{ɏ}$ and ɏ is a positive even integer by *Proposition 6* (II) and *Proposition 7* (II).

Proposition 8. Let $\mathfrak{I}_G$ be an $\mathcal{SS}$. Then,

I. $\mathfrak{I}_G \otimes_{s'/l} \emptyset_G = \emptyset_G$, where $|G| = \text{ɏ}$ and ɏ is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/l} \emptyset_G = U_G$, where $|G| = \text{ɏ}$ and ɏ is a positive even integer.

Proof: let $\mathfrak{I}_G$ be an $\mathcal{SS}$.

I. Suppose that $|G| = \text{ɏ}$, where ɏ is a positive odd integer. Then, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} \emptyset_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset_G{}^c(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U_G(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U)$$

$$= \emptyset_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \emptyset_G = \emptyset_G$.

I. Suppose that $|G| = \text{ɏ}$, where ɏ is a positive even integer. Then, for all $\varphi \in G$,

$$(\mathfrak{I}_G \otimes_{s'/l} \emptyset_G)(\varphi) = \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup \emptyset_G{}^c(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U_G(\eta))$$

$$= \coprod_{\varphi=\varrho\eta} (\mathfrak{I}_G(\varrho) \cup U)$$

$$= U_G(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} \emptyset_G = U_G$. □

Note: By *Proposition 8* (I), $\emptyset_G$ is the right absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ɏ$ and ɏ is a positive odd integer; however, $\emptyset_G$ is not the right absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ɏ$ and ɏ is a positive odd integer. In fact, consider the group $G = \{a. b. c\}$ with the following operation:

Table 2. Non-absorbing property in soft symmetric difference complement-λ product over group $G = \{a, b, c\}$.

0	a	b	c
a	a	b	c
b	b	c	a
c	c	a	b

Let $\Im_G$ be an SS over $U = D_2 = \{< x. y >: x^2 = y^2 = e. xy = yx\} = \{e. x. y. yx\}$ as follows:

$$\Im_G = \{(a. \{e. y\}). (b. \{x\}). ((c. \{y\})\}.$$

Then,

$$\emptyset_G \otimes_{s'/l} \Im_G = \{(a. \{e, x\}). (b. \{e, x\}). (c. \{e, x\})\} \neq \emptyset_G.$$

Thus, $\emptyset_G$ is not the absorbing element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ɏ$ and ɏ is a positive odd integer.

Proposition 9. Let $\Im_G$ be a constant SS. Then,

I. $\emptyset_G \otimes_{s'/l} \Im_G = \Im_G$, where $|G| = ɏ$ and ɏ is a positive odd integer.

II. $\emptyset_G \otimes_{s'/l} \Im_G = U_G$, where $|G| = ɏ$ and ɏ is a positive even integer.

Proof: let $\Im_G$ be a constant SS such that, for all $\varphi \in G$, $\Im_G(\varphi) = Ɲ$, where Ɲ is a fixed set.

Let $|G| = ɏ$, where ɏ is a positive odd integer. Then, for all $\varphi \in G$,

$$\left(\emptyset_G \otimes_{s'/l} \Im_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\emptyset_G(\varrho) \cup \Im_G{}^c(\eta)\right) = \coprod_{\varphi = \varrho\eta} \left(\emptyset \cup \Im_G{}^c(\eta)\right) = \Im_G(\varphi).$$

Thereby, $\emptyset_G \otimes_{s'/l} \Im_G = \Im_G$.

Let $|G| = ɏ$, where ɏ is a positive even integer. Then, for all $\varphi \in G$,

$$\left(\emptyset_G \otimes_{s'/l} \Im_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\emptyset_G(\varrho) \cup \Im_G{}^c(\eta)\right) = \coprod_{\varphi = \varrho\eta} \left(\emptyset \cup \Im_G{}^c(\eta)\right) = U_G(\varphi).$$

Thereby, $\emptyset_G \otimes_{s'/l} \Im_G = U_G$. □

Remark 5. $\emptyset_G$ is the right identity element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ɏ$ and ɏ is a positive odd integer by *Proposition 9* (I). In fact, $\emptyset_G$ is not the identity element of the soft symmetric difference complement-lambda product in $S_G(U)$, where $|G| = ɏ$ and ɏ is a positive odd integer by *Proposition 8* (I).

Proposition 10. Let $\Im_G$ be a constant SS. Then,

I. $\Im_G \otimes_{s'/l} \Im_G{}^c = \Im_G{}^c$, where $|G| = ɏ$ and ɏ is a positive odd integer.

II. $\Im_G \otimes_{s'/l} \Im_G{}^c = U_G$, where $|G| = ɏ$ and ɏ is a positive even integer.

Proof: let $\Im_G$ be a constant SS such that, for all $\varphi \in G$, $\Im_G(\varphi) = Ɲ$, where Ɲ is a fixed set.

I. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive odd integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left(\Im_G \otimes_{s'/l} {\Im_G}^c\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup ({\Im_G}^c)^c(\eta)\right) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup \Im_G(\eta)\right) = {\Im_G}^c(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} {\Im_G}^c = {\Im_G}^c$.

II. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive even integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left(\Im_G \otimes_{s'/l} {\Im_G}^c\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup ({\Im_G}^c)^c(\eta)\right) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup \Im_G(\eta)\right) = U_G(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} {\Im_G}^c = U_G$.

Proposition 11. Let $\Im_G$ be a constant $\mathcal{SS}$. Then,

I. ${\Im_G}^c \otimes_{s'/l} \Im_G = \Im_G$, where $|\mathrm{G}| = ӌ$ and ӌ is a positive odd integer.

II. ${\Im_G}^c \otimes_{s'/l} \Im_G = U_G$, where $|\mathrm{G}| = ӌ$ and ӌ is a positive even integer.

Proof: let $\Im_G$ be a constant $\mathcal{SS}$ such that, for all $\varphi \in \mathrm{G}$, $\Im_G(\varphi) = Ɲ$, where Ɲ is a fixed set.

I. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive odd integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left({\Im_G}^c \otimes_{s'/l} \Im_G\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left({\Im_G}^c(\varrho) \cup {\Im_G}^c(\eta)\right) = \Im_G(\varphi).$$

Thereby, ${\Im_G}^c \otimes_{s'/l} \Im_G = \Im_G$.

II. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive even integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left({\Im_G}^c \otimes_{s'/l} \Im_G\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left({\Im_G}^c(\varrho) \cup {\Im_G}^c(\eta)\right) = U_G(\varphi).$$

Thereby, ${\Im_G}^c \otimes_{s'/l} \Im_G = U_G$.

Proposition 12. Let $\Im_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s such that $\mathfrak{q}_G \tilde{\subseteq}_A \Im_G$. Then, $\Im_G \otimes_{s'/l} \mathfrak{q}_G = U_G$.

I. $\Im_G \otimes_{s'/l} \mathfrak{q}_G = \emptyset_G$, where $|\mathrm{G}| = ӌ$ and ӌ is a positive odd integer.

II. $\Im_G \otimes_{s'/l} \mathfrak{q}_G = U_G$, where $|\mathrm{G}| = ӌ$ and ӌ is a positive even integer.

Proof: let $\Im_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s and $\mathfrak{q}_G \tilde{\subseteq}_A \Im_G$. Then, $\mathfrak{q}_G(\varrho) \subseteq \Im_G(\eta)$, for each $\varrho, \eta \in \mathrm{G}$.

I. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive odd integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left(\Im_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup {\mathfrak{q}_G}^c(\eta)\right) = \emptyset_G(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} \mathfrak{q}_G = \emptyset_G$. Here note that, $\Im_G(\varrho) \cup {\mathfrak{q}_G}^c(\eta) = \left(\mathfrak{q}_G(\eta) \setminus \Im_G(\varrho)\right)'$, for all $\varrho, \eta \in \mathrm{G}$.

II. Let $|\mathrm{G}| = ӌ$, where ӌ is a positive even integer. Then, for all $\varphi \in \mathrm{G}$,

$$\left(\Im_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi=\varrho\eta} \left(\Im_G(\varrho) \cup {\mathfrak{q}_G}^c(\eta)\right) = U_G(\varphi).$$

Thereby, $\Im_G \otimes_{s'/l} \mathfrak{q}_G = U_G$.

Proposition 13. Let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s such that $\mathfrak{q}_G \cong_S \mathfrak{I}_G$. Then,

I. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = \emptyset_G$, where $|G| = \mathrm{y}$ and y is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = U_G$, where $|G| = \mathrm{y}$ and y is a positive even integer.

Proof: the proof is similar to the proof of *Proposition 12*.

Proposition 14. Let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s such that $\mathfrak{I}_G \cong_S (\mathfrak{q}_G)^c$. Then,

I. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = \mathfrak{q}_G$, where $|G| = \mathrm{y}$ and y is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = U_G$, where $|G| = \mathrm{y}$ and y is a positive even integer.

Proof: let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s.

I. Let $\mathfrak{I}_G \cong_S (\mathfrak{q}_G)^c$, $|G| = \mathrm{y}$, where y is a positive odd integer. Then, for all $\varphi \in G$, $\mathfrak{I}_G(\varphi) = \mathrm{N}$, $\mathfrak{q}_G(\varphi) = \mathrm{D}$, where N and D are two fixed sets and $\mathrm{N} \subseteq \mathrm{D}'$. Hence, for all $\varphi \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\mathfrak{I}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right) = \mathfrak{q}_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = \mathfrak{q}_G$.

II. Let $\mathfrak{I}_G \cong_S (\mathfrak{q}_G)^c$, $|G| = \mathrm{y}$, where y is a positive even integer. Then, for all $\varphi \in G$, $\mathfrak{I}_G(\varphi) = \mathrm{N}$, $\mathfrak{q}_G(\varphi) = \mathrm{D}$, where N and D are two fixed sets and $\mathrm{N} \subseteq \mathrm{D}'$. Hence, for all $\varphi \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\mathfrak{I}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right) = U_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = U_G$.

Proposition 15. Let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s such that $(\mathfrak{q}_G)^c \cong_S \mathfrak{I}_G$. Then,

I. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = \mathfrak{I}_G{}^c$, where $|G| = \mathrm{y}$ and y is a positive odd integer.

II. $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = U_G$, where $|G| = \mathrm{y}$ and y is a positive even integer.

Proof: let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s.

I. Let $(\mathfrak{q}_G)^c \cong_S \mathfrak{I}_G$, $|G| = \mathrm{y}$, where y is a positive odd integer. Then, for all $\varphi \in G$, $\mathfrak{I}_G(\varphi) = \mathrm{N}$, $\mathfrak{q}_G(\varphi) = \mathrm{D}$, where N and D are two fixed sets and $\mathrm{D}' \subseteq \mathrm{N}$. Hence, for all $\varphi \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\mathfrak{I}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right) = \mathfrak{I}_G{}^c(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = \mathfrak{I}_G{}^c$.

II. Let $(\mathfrak{q}_G)^c \cong_S \mathfrak{I}_G$, $|G| = \mathrm{y}$, where y is a positive even integer. Then, for all $\varphi \in G$, $\mathfrak{I}_G(\varphi) = \mathrm{N}$, $\mathfrak{q}_G(\varphi) = \mathrm{D}$, where N and D are two fixed sets and $\mathrm{D}' \subseteq \mathrm{N}$. Hence, for all $\varphi \in G$,

$$\left(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G\right)(\varphi) = \coprod_{\varphi = \varrho\eta} \left(\mathfrak{I}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right) = U_G(\varphi).$$

Thereby, $\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G = U_G$.

Proposition 16. Let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s. Then, $\left(\mathfrak{I}_G \otimes_{s'/l} \mathfrak{q}_G\right)^c = \mathfrak{I}_G \otimes_{s/g} \mathfrak{q}_G$.

Proof: let $\mathfrak{I}_G$ and $\mathfrak{q}_G$ be two $\mathcal{SS}$s. Then, for all $\varphi \in G$,

$$\left(\mathfrak{J}_G \otimes_{s'/l} \mathfrak{q}_G\right)^c(\varphi) = \left(\coprod_{\varphi=\varrho\eta} \left(\mathfrak{J}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right)\right)'$$

$$= \underset{\varphi=\varrho\eta}{\triangle} \left(\mathfrak{J}_G(\varrho) \cup \mathfrak{q}_G{}^c(\eta)\right)'$$

$$= \underset{\varphi=\varrho\eta}{\triangle} \left(\mathfrak{J}_G{}^c(\varrho) \cap \mathfrak{q}_G(\eta)\right)$$

$$= \left(\mathfrak{J}_G \otimes_{s/g} \mathfrak{q}_G\right)(\varphi).$$

Thereby, $\left(\mathfrak{J}_G \otimes_{s'/l} \mathfrak{q}_G\right)^c = \mathfrak{J}_G \otimes_{s/g} \mathfrak{q}_G$. □

4 | Conclusion

This study proposes the soft symmetric difference complement–lambda product, a novel binary operation defined on soft sets whose parameter sets are groups. The operation is analyzed with generalized notions of soft equality. The investigation further examines its compatibility with null and absolute soft sets. Basic algebraic properties—including closure, associativity, commutativity, and idempotency—are systematically investigated, with detailed consideration of identity, inverse, and absorbing elements. Fundamentally, this operation provides a foundational step toward a generalized soft group theory. Consequently, this work significantly enriches the theoretical foundations of soft set theory.

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Data Availability

All data are included in the text.

Conflicts of Interest

The authors stated that there are no conflicts of interest regarding the publication of this article.

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